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Neutrino masses and μ terms in a supersymmetric extra U(1) model

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Abstract

We propose a supersymmetric extra U(1) model, which can generate small neutrino masses and necessary μ terms, simultaneously. Fields including quarks and leptons are embedded in three **27**s of E_6 in a different way among generations. The model has an extra U(1) gauge symmetry at TeV regions, which has discriminating features from other models studied previously. Since a neutrino mass matrix induced in the model has a constrained texture with limited parameters, it can give a prediction. If we impose neutrino oscillation data to fix those parameters, a value of $\sin\theta_{13}$ can be determined. We also discuss several phenomenological features which are discriminated from the ones of the MSSM.

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1 Introduction

Recent experimental studies on neutrinos, the cosmic microwave background and the large scale structure of the universe have suggested the existence of neutrino masses [1–4] and dark matter [5, 6]. These give us strong motivation to examine various possibilities for the extension of the standard model (SM). If new neutral fields with suitable interactions are added to the SM, small neutrino masses can be generated and the origin of dark matter may also be explained, simultaneously. In fact, the seesaw mechanism has been well known as such a typical example [7–9][d] and also the radiative seesaw mechanism recently attracts much attention [12]. In the latter scenario, especially, new neutral fields added to generate neutrino masses can behave as cold dark matter as long as it is the lightest stable field among newly introduced ones [12–28]. However, the gauge hierarchy problem is put aside in the most study of these models.

On the other hand, in the minimal supersymmetric SM (MSSM) which is motivated to solve the gauge hierarchy problem, a dark matter candidate is automatically built in the model as the lightest neutralino as long as R-parity is assumed to be conserved [29]. However, even if singlet chiral superfields are added as right-handed neutrinos in this model, small neutrino masses may not be explained without assuming the existence of an intermediate scale of $O(10^{11-13})$ GeV as the origin of the right-handed neutrino mass.[e] Since the origin of that scale is not explained without further extension of the model, we have to consider other additional fields as in the grand unified models. It is an interesting subject to find such a consistent extension without inducing other phenomenological problems. Proton stability, doublet-triplet splitting and also the μ problem are important issues to be answered in that consideration.[f]

In this paper we study these subjects from a view point of the neutrino mass generation in the framework of a supersymmetric extra U(1) model. In general, models with extra U(1) gauge symmetry are annoyed with gauge anomaly problem. However, if we identify this U(1) symmetry with the ones derived from E_6 [36], the anomaly problem can be solved

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[d]In the νMSM proposed in [10, 11] which follows ordinary seesaw, since right-handed neutrinos are assumed to be lighter than the weak scale, the lightest right-handed neutrino can behave as dark matter.

[e]If we consider the supersymmetric extension of the radiative seesaw model [30], we can have the right-handed neutrinos with $O(1)$ TeV masses.

[f]The relation between the small neutrino masses and the μ problem has been discussed in [31–35], for example.

automatically. It occurs as long as the full members of the irreducible representation of E_6 are contained in the model. It is also well known that such supersymmetric models can appear as the low energy effective models of heterotic string with Wilson line breaking [37, 38]. An interesting point of these models is that the required low energy gauge symmetry can be derived from E_6 without any superfields of higher dimensional representations. Thus, the models can be constructed by strictly restricted fields only.

This type of model has a lot of interesting features. One of them is that the models can have two U(1) gauge symmetries in addition to the $U(1)_Y$ in the SM [38–40]. They can play important roles phenomenologically at TeV scales or at intermediate scales of $O(10^{11-13})$ GeV. For example, the μ problem has been shown to be solved elegantly by using this TeV scale extra U(1) symmetry [31, 41]. On the other hand, the intermediate scale can be introduced as the breaking scale of another extra U(1) symmetry through a D-flat direction. This scale can be related to the masses of the right-handed neutrinos [31]. It can also make extra matter fields heavy enough to decouple from low energy phenomena [32, 39, 40]. However, if we set the model so as to use this intermediate scale for the mass generation of the right-handed neutrinos, we can not make extra color triplet fields and extra Higgs doublet fields heavy enough. The extra U(1) symmetry constrained as a subgroup of E_6 forbids interaction terms required to make such fields heavy. Since these extra fields couple with quarks and leptons, dangerous couplings which induce proton decay and flavor changing neutral current (FCNC) remains in the low energy regions in general.[g]

In order to improve the difficulty of this type of extra U(1) model, we propose a novel modification. In the models considered previously, the fields are assigned to a fundamental representation **27** of E_6 in the same way among the generation. In this paper, we modify it among the generation [32] by imposing discrete symmetry on the superpotential of the model. Under this setting, we show that this new model can solve the above mentioned various problems in the ordinary model of this type, simultaneously.[h] On the basis of this

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[g]In spite of this fault, this type of models have various interesting features. Study on those points can be found in [42–73], for example.

[h]Only a type of extra U(1) at low energy regions has been known to generate small neutrino masses due to a high-scale seesaw mechanism in the E_6 framework. It corresponds to a case with $\theta = \arctan\sqrt{15}$ if we define this extra U(1)′ as U(1)′=$U(1)_\chi \cos\theta + U(1)_\psi \sin\theta$. We note that our U(1) symmetry discussed in this paper corresponds to $U(1)_\chi$, which gives a new possibility for neutrino mass generation.

model, we derive a neutrino mass matrix and analyze it by using the neutrino oscillation data. Other phenomenological features which are discriminated from the ones of the MSSM are also discussed.

The remaining parts are organized as follows. In section 2 we define our model and discuss the discrete symmetry considered in the model. We also address intermediate scales and explain its possible role in the model. In section 3 we fix the effective model at the TeV regions and study its various phenomenological features. Neutrino mass generation in the model and results derived from it are investigated in detail. In section 4 we summarize the paper.

2 A model based on E_6

2.1 Field contents and symmetry

We consider a model which is expected to be derived as an effective model of string inspired E_6 models through Wilson line breaking [37,38]. Gauge symmetry is $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)^3$ which is expected to be induced from E_6 due to the Wilson line breaking. Massless chiral superfields are composed of three of the E_6 fundamental representation $\mathbf{27}$ and a part of the vector like pair $\mathbf{27} + \overline{\mathbf{27}}$ [38–40]. A fundamental representation $\mathbf{27}$ of E_6 can be decomposed under the above mentioned low energy gauge symmetry as shown in Table 1. As a remaining content of the vector like pair $\mathbf{27} + \overline{\mathbf{27}}$, we take $\mathcal{A}_7 + \bar{\mathcal{A}}_7$.[i] Features of the model crucially depend on the way to embed the physical fields in A_1 - A_{11} of $\mathbf{27}$. Although this embedding is usually done in the same way for every generation, we adopt here a twisted field assignment as shown in Table 1, where the fields are embedded differently among generations [32].

Gauge invariant renormalizable superpotential is given by

$$
\begin{aligned}
W_1 \;=\;& A_1 A_2 A_8 + A_1 A_3 A_9 + A_4 A_5 A_9 + A_4 A_6 A_8 + A_7 A_8 A_9 + A_7 A_{10} A_{11} \\
&+\; A_1 A_1 A_{10} + A_1 A_4 A_{11} + A_2 A_3 A_{11} + A_2 A_5 A_{10} + A_3 A_6 A_{10} \\
&+\; \mathcal{A}_7 A_8 A_9 + \mathcal{A}_7 A_{10} A_{11},
\end{aligned}
\tag{1}
$$

where Yukawa couplings and generation indices are abbreviated. These terms generally exist unless some additional symmetry forbids them. Since the terms in the second line

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[i]This type of field content has been suggested to be induced through Wilson line breaking [39,40].

are dangerous for proton stability, a part of those terms is required to disappear from the low energy effective superpotential by imposing some additional symmetry or by making extra fields sufficiently heavy as a result of symmetry breaking at some high energy scales. As easily seen from Table 1, the model has three color triplet pairs $(g, \bar{g})$ and two extra Higgs doublet pairs (H_u, H_d) beyond the MSSM contents at this stage.

It is important to note that there also appear gauge invariant nonrenormalizable terms

$$W_1^{NR} = \sum_{(a,b,c)} \left(\frac{\mathcal{A}_7 \bar{\mathcal{A}}_7}{M_{\rm pl}^2} \right)^{n_{abc}} A_a A_b A_c \equiv \sum_{(a,b,c)} \epsilon^{n_{abc}} A_a A_b A_c, \qquad (A_a A_b A_c \in W_1), \quad (2)$$

where $M_{\rm pl}$ is the Planck mass and n_{abc} is a positive integer to be determined for each term $A_a A_b A_c$ in W_1 independently. The value of n_{abc} depends on the additional symmetry considered in the model. Some of these can cause important contributions to the low energy effective models if both $\mathcal{A}_7$ and $\bar{\mathcal{A}}_7$ obtain large vacuum expectation values (VEVs). In particular, the terms in W_1^{NR} corresponding to the last two terms in W_1 should be taken into account since they are relevant to the mass of the above mentioned extra fields. These points will be discussed in the next subsection.

Now we consider the model with $Z_2 \times Z_4$ as the additional symmetry.[j] Its charge assignment for each chiral superfield in **27** are also shown in Table. 1. The Z_2 symmetry will be identified with R parity. For $\mathcal{A}_7$ and $\bar{\mathcal{A}}_7$, we assign them the Z_2 parity and the Z_4 charge as $(+, +1)$ and $(+, 0)$, respectively. The allowed terms in superpotential W_1 under this discrete symmetry can be restricted as follows,

$$
\begin{aligned}
W_2 &= Q_i \bar{U}_j H_{u_2} + Q_i \bar{D}_j H_{d_2} + L_\alpha \bar{E}_j H_{d_2} + L_3 \bar{E}_j H_{d_3} + S_2 H_{u_2} H_{d_2} + S_3 H_{u_2} H_{d_3} \\
&+ S_2 g_3 \bar{g}_3 + \bar{N}_3 g_1 \bar{g}_2 + Q_i \bar{g}_3 H_{d_3} + g_3 \bar{D}_i S_3 + g_1 \bar{D}_i S_3 + g_3 \bar{D}_i \bar{N}_2 \\
&+ L_\alpha S_3 H_{u_3} + L_3 S_2 H_{u_3} + L_\alpha \bar{N}_2 H_{u_3} + \bar{N}_1 H_{u_1} H_{d_3} + \bar{N}_2 H_{u_2} H_{d_3} \\
&+ \bar{N}_3 H_{u_2} H_{d_1} + \mathcal{A}_7 H_{u3} H_{d_1} + Q_i Q_j g_2 + \bar{U}_i \bar{E}_j g_2 + \mathcal{A}_7 g_\alpha \bar{g}_\alpha, \quad (3)
\end{aligned}
$$

where Yukawa couplings are omitted again. Generation indices are labeled by Greek and Latin characters $\alpha = 1, 2$ and $i = 1, 2, 3$. Nonrenormalizable terms allowed by the same

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[j] Since we suppose the Wilson line breaking of E_6, this discrete symmetry should be consistent with the gauge symmetry SU(3)×SU(2)× U(1)^3 but not with E_6. We may be able to find relations between this symmetry and the discrete symmetry which is used to define the multiply connected manifold as the basis of Wilson line breaking as shown in [39, 40]. It is likely that they are identified each other and in that case the present discrete symmetry is considered to be built in the model originally.

	SU(3)×SU(2)×U(1)^3	Field assignment($Z_2 \times Z_4$)		Light fields
A_1	$(\mathbf{3},\mathbf{2},\frac{1}{6},1,-1)$	$Q_\alpha\ (-,+1)$	$Q_3\ (-,+1)$	Q_i
A_2	$(\mathbf{3}^*,\mathbf{1},-\frac{2}{3},1,-1)$	$\bar{U}_\alpha\ (-,+2)$	$\bar{U}_3\ (-,+2)$	$\bar{U}_i$
A_3	$(\mathbf{3}^*,\mathbf{1},\frac{1}{3},1,3)$	$\bar{D}_\alpha(-,0)$	$\bar{D}_3\ (-,0)$	$\bar{D}_i$
A_4	$(\mathbf{1},\mathbf{2},-\frac{1}{2},1,3)$	$L_\alpha\ (-,+1)$	$H_{d3}\ (+,+2)$	$L_\alpha,\ H_{d3}$
A_5	$(\mathbf{1},\mathbf{1},1,1,-1)$	$\bar{E}_\alpha\ (-,0)$	$\bar{E}_3\ (-,0)$	$\bar{E}_i$
A_6	$(\mathbf{1},\mathbf{1},0,1,-5)$	$\bar{N}_{\frac{1}{2}}\ (+,{}^{\ 0}_{+1})$	$S_3\ (+,+1)$	$\bar{N}_\alpha,\ S_3$
A_7	$(\mathbf{1},\mathbf{1},0,4,0)$	$S_{\frac{1}{2}}\ (+,{}^{+2}_{\ 0})$	$\bar{N}_3\ (-,+2)$	S_2
A_8	$(\mathbf{1},\mathbf{2},-\frac{1}{2},-2,2)$	$H_{u\,\frac{1}{2}}\ (+,{}^{+2}_{+1})$	$H_{u3}\ (-,+2)$	$H_{u\alpha}$
A_9	$(\mathbf{1},\mathbf{2},-\frac{1}{2},-2,-2)$	$H_{d\,\frac{1}{2}}\ ({}^{-}_{+},{}^{+1}_{+3})$	$L_3\ (-,+2)$	$H_{d2},\ L_3$
A_{10}	$(\mathbf{3},\mathbf{1},-\frac{1}{3},-2,2)$	$g_{\frac{1}{2}}\ ({}^{-}_{+},{}^{+1}_{+2})$	$g_3\ (-,+3)$	g_3
A_{11}	$(\mathbf{3}^*,\mathbf{1},\frac{1}{3},-2,-2)$	$\bar{g}_{\frac{1}{2}}\ ({}^{-}_{+},{}^{+2}_{+1})$	$\bar{g}_3\ (-,+1)$	$\bar{g}_3$

Table 1 Decomposition of $\mathbf{27}$ and the field assignment. Abelian charges Y, Q_ψ and Q_χ for $U(1)_Y \times U(1)_\psi \times U(1)_\chi$ are listed as $\sqrt{\frac{5}{3}}Y$, $2\sqrt{10}Q_\psi$ and $2\sqrt{6}Q_\chi$, respectively [36]. Greek and Latin indices of the fields stand for the generation. On the discrete symmetry, we show the Z_2 parity and the charge for Z_4 of each field, respectively.

symmetry in W_1^{NR} are easily found to be restricted to

$$
\begin{aligned}
W_2^{NR} =\ & \epsilon^2 \mathcal{A}_7 H_{u_1} H_{d_2} + \epsilon^3 \mathcal{A}_7 H_{u_2} H_{d_2} + \epsilon^3 \mathcal{A}_7 L_3 H_{u_3} + \epsilon^3 \mathcal{A}_7 g_3 \bar{g}_3 + \epsilon \mathcal{A}_7 g_1 \bar{g}_3 + \epsilon^2 \mathcal{A}_7 g_3 \bar{g}_1 \\
& + \epsilon \big(H_{d_3} \bar{E}_i H_{d_1} + L_\alpha \bar{N}_1 H_{u_3} + H_{d_3} \bar{N}_1 H_{u_2} + S_1 H_{u_1} H_{d_2} + S_2 H_{u_3} H_{d_1} \\
& + \bar{N}_3 H_{u_3} H_{d_2} + S_2 g_\alpha \bar{g}_\alpha + Q_i L_\alpha \bar{g}_2 + \bar{U}_i \bar{D}_j \bar{g}_2 + \bar{D}_i \bar{N}_1 g_3 \big) + O(\epsilon^2).
\end{aligned}
\tag{4}
$$

In eq. (4), we list up all terms corresponding to the ones in W_1 which include $\mathcal{A}_7$. For other terms, we write only the $O(\epsilon)$ terms in eq. (3). The reason for this is that the VEV of $\mathcal{A}_7$ is considered large enough but $\epsilon \ll 1$, as shown in the next subsection.

As found in these superpotentials W_2 and W_2^{NR}, there still remain the couplings among extra colored fields and quarks which are dangerous for proton stability. Moreover, small neutrino masses can not be explained only from these. The existence of extra fields could also spoil gauge coupling unification. Because of these reasons, it seems to be favorable that a part of these extra fields becomes heavy through some symmetry breaking at a much higher energy scale than the weak scale. If the fields contributing to neutrino mass generation become heavy enough due to the same symmetry breaking, the smallness of

neutrino masses can also be explained. In the next part, we address the possibility to cause this symmetry breaking at a desirable high energy scale.

2.2 An intermediate scale induced by a D-flat direction

Problems addressed at the end of the last part can be solved if scalar components of a vector like pair $\mathcal{A}_7 + \bar{\mathcal{A}}_7$ have large VEVs. The lowest order invariant superpotential for $\mathcal{A}_7$ and $\bar{\mathcal{A}}_7$ is written as

$$W_\mathcal{A} = \frac{c}{M_{\rm pl}^5} \left(\mathcal{A}_7 \bar{\mathcal{A}}_7 \right)^4, \tag{5}$$

where the coupling constant c is naturally considered to be $O(1)$. Since $\langle \mathcal{A}_7 \rangle = \langle \bar{\mathcal{A}}_7 \rangle = \phi$ gives a D-flat direction of extra U(1) gauge symmetries, minimum points of the scalar potential for these are expected to appear along this direction [38–40]. On these points, both the extra gauge symmetry U(1)$_\chi$ and the Z_2 symmetry remain unbroken. Although Z_4 is spontaneously broken, an associated domain wall problem can not be serious in this case. Since this symmetry breaking occurs at sufficiently high energy scale as seen below, inflation is expected to occur after it to resolve this problem. The remaining U(1)$_\chi$ is expected to give a solution to the μ problem at TeV scales in the way as suggested in [31]. Moreover, since the Z_2 parity of the SM contents are assigned as even, the lightest Z_2 odd field is stable to be a dark matter candidate.

The scalar potential derived from $W_\mathcal{A}$ along this D-flat direction is expressed by the VEV ϕ as

$$V \simeq \frac{32}{M_{\rm pl}^{10}} |\phi|^{14} - 2m_s^2 |\phi|^2, \tag{6}$$

where m_s stands for the soft supersymmetry breaking mass of $O(1)$ TeV. By minimizing the scalar potential V, we can determine a value of ϕ as

$$|\phi| \simeq \left(0.2 M_{\rm pl}^5 m_s \right)^{\frac{1}{6}}. \tag{7}$$

This gives $|\phi| \sim 2 \times 10^{16}$ GeV and then ϵ defined in eq. (2) is estimated as $\epsilon \sim 10^{-6}$. Since fermionic components of $\mathcal{A}_7$ and $\bar{\mathcal{A}}_7$ mix with a broken U(1)$_\psi$ gaugino, their mass eigenvalues are expected to be $O(g_\psi |\phi|)$ where g_ψ is a gauge coupling constant of U(1)$_\psi$.

The symmetry breaking at this scale controls the massless field contents which constitute the low energy effective model through the couplings with $\mathcal{A}_7$. This is found in W_2. Although the model has three pairs of Higgs doublets originally, only a part of them can

remain massless. In fact, a gauge invariant coupling $\mathcal{A}_7 H_{u3} H_{d_1}$ can induce mass terms between Higgs chiral superfields H_{u_3} and H_{d_1}. On the other hand, since $\mathcal{A}_7$ has also gauge invariant couplings with extra colored fields as $\mathcal{A}_7 g_\alpha \bar{g}_\alpha$,[k] the VEV ϕ generates large masses for them. Thus, only one pair of extra color triplets remains almost massless. We find that the mass of the states dominated by g_3 or $\bar{g}_3$ is $O(\epsilon^3 |\phi|)$ by removing the mixings $\epsilon^2 \mathcal{A}_7 g_3 \bar{g}_1$ and $\epsilon \mathcal{A}_7 g_1 \bar{g}_3$ appearing in W_2^{NR}. The VEV ϕ also makes some of singlets $\bar{N}_i$ and S_i very heavy through another gauge invariant nonrenormalizable coupling

$$\frac{1}{M_{\mathrm{pl}}} (\bar{\mathcal{A}}_7 \mathcal{A}_7)^2. \tag{8}$$

This coupling is controlled by the discrete symmetry Z_4 and generates the mass of $O(|\phi|^2/M_{\mathrm{pl}})$ for $\bar{N}_3$ and S_1, which may play a role of heavy right-handed neutrinos with the mass of $O(10^{13})$ GeV [31].

Here, it is worthy to note the magnitude of the couplings with $\mathcal{A}_7$ in other terms, which are forbidden by Z_4 as the renormalizable couplings in W_2 but appear as the invariant nonrenormalizable couplings in W_2^{NR}. Since $\epsilon^3 \mathcal{A}_7 H_{u_2} H_{d_2}$ and $\epsilon^3 \mathcal{A}_7 H_{u_3} L_3$ are sufficiently suppressed, they can be neglected in the following discussion. On the other hand, since $\epsilon^2 \mathcal{A}_7 H_{u_1} H_{d_2}$ induces a weak scale mass term for H_{u_1} and H_{d_2} if Yukawa coupling is a little small, it can play an important role in the low energy phenomenology.

Taking account of the facts discussed above, we see that the massless chiral superfields in the model are confined to the ones listed in the last column of Table 1. At the scale of ϕ, they are composed of the MSSM contents and also the extra chiral superfields which are summarized as $(\mathbf{5},\ \bar{\mathbf{5}}) + 4(\mathbf{1})$ of SU(5). These massless contents keep the unification of the MSSM gauge coupling constants at $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV, since the addition of $\mathbf{5} + \bar{\mathbf{5}}$ to the MSSM changes the one-loop β-function of each MSSM gauge coupling constant in the same way. It is noticeable that the symmetry breaking scale obtained above happens to coincide with the scale of this M_{GUT}.

3   A low energy effective model

Phenomenological features of the low energy effective model obtained from the discussion in the previous section are determined by the superpotential which is composed of the

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[k] This coupling of extra colored fields with $\mathcal{A}_7$ may be relevant to a solution for the strong CP problem [74, 75].

light chiral superfields,[1]

$$
\begin{aligned}
W_3 &= h_U^{ij} Q_i \bar{U}_j H_{u_2} + h_D^{ij} Q_i \bar{D}_j H_{d_2} + h_E^{\alpha j} L_\alpha \bar{E}_j H_{d_2} + h_E^{3j} L_3 \bar{E}_j H_{d_3} \\
&+ \lambda_1 S_2 H_{u_2} H_{d_2} + \lambda_2 S_3 H_{u_2} H_{d_3} + k S_2 g_3 \bar{g}_3 + h_{\bar{g}}^i Q_i \bar{g}_3 H_{d3} + h_g^i g_3 \bar{D}_i S_3 \\
&+ f_\alpha L_\alpha S_3 H_{u3} + f_3 L_3 S_2 H_{u3} + f'_\alpha L_\alpha \bar{N}_2 H_{u3} \\
&+ \kappa_1 \bar{N}_1 H_{u_1} H_{d_3} + \kappa_2 \bar{N}_2 H_{u_2} H_{d_3} + \kappa'_i g_3 \bar{D}_i \bar{N}_1.
\end{aligned}
\tag{9}
$$

This superpotential W_3 includes necessary terms to realize the favorable structure of the MSSM and also the important terms for the neutrino mass generation. It should be noted that the third line in W_3 contains the terms which include heavy chiral superfields relevant to the neutrino mass generation. As soft supersymmetry breaking parameters, we introduce the soft masses m_s^2 for scalar components of all chiral superfields, and a trilinear scalar coupling A with mass dimension one which appears for each term in W_3, and the soft masses of gauginos $\tilde{W}_i$, $\tilde{B}$, $\tilde{\chi}$ for each gauge factor group SU(2), U(1)$_Y$, U(1)$_\chi$ and also gluinos. All other mass scales in the model are considered to be radiatively generated from these soft supersymmetry breaking parameters. Now, we address several interesting features of this model in the following parts.

3.1 μ terms and quark/lepton masses

The model has two kinds of μ term. Following the radiative symmetry breaking scenario based on the renormalization group equations (RGEs) for the soft scalar masses of the singlet scalars $S_{2,3}$, they can obtain the VEVs as follows,

$$
\langle S_2 \rangle = u, \qquad \langle S_3 \rangle = u'.
\tag{10}
$$

This is expected to occur since these singlet scalars have couplings with the colored fields and then the squared masses of $S_{2,3}$ can take negative values at weak scales [31, 41]. As a result of these VEVs, two μ terms are generated from the $\lambda_{1,2}$ terms as $\mu = \lambda_1 u$ and $\mu' = \lambda_2 u'$. Since H_{u_2} has a large Yukawa coupling with top quark, it is also expected to have a VEV as usual. Thus, if we note the existence of effective μ terms and remind the experience in the MSSM, the vacuum structure at TeV regions is considered to be fixed

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[1] In the following parts, the scalar component is expressed by the same character as the chiral superfield and a tilde is put on the character for the fermionic component.

by

$$\langle H_{u_2} \rangle = v_2, \quad \langle H_{u_3} \rangle = 0, \quad \langle H_{d_2} \rangle = v_{1a}, \quad \langle H_{d_3} \rangle = v_{1b}, \qquad (11)$$

where we define $v_1^2 = v_{1a}^2 + v_{1b}^2$ and $\tan\beta = v_2/v_1$. The Z_2 symmetry remains as an exact symmetry in this vacuum. It is identified with the R-parity, since the charge can be assigned such that the SM contents are even and their superpartners are odd under it. It is useful to note that the usual R-parity violating terms allowed by the MSSM gauge structure are also forbidden at the level of W_2 through the existence of the extra U(1) symmetries. This Z_2 symmetry can guarantee the stability of the lightest particle with odd parity.

Under this vacuum, the masses of quarks and charged leptons are generated as

$$(m_u)^{ij} = h_U^{ij} v_2, \quad (m_d)^{ij} = h_D^{ij} v_{1a}, \quad (m_e)^{ij} = \begin{pmatrix} h_E^{\alpha 1} v_{1a} & h_E^{\alpha 2} v_{1a} & h_E^{\alpha 3} v_{1a} \\ h_E^{31} v_{1b} & h_E^{32} v_{1b} & h_E^{33} v_{1b} \end{pmatrix}. \qquad (12)$$

Neutrino masses are discussed in the next part. The charged lepton mass matrix is generated by the couplings with two Higgs doublets $H_{d_{2,3}}$. This could induce dangerous lepton flavor violating processes in principle. However, it can be easily escaped as long as the Yukawa coupling constants satisfy simple relations given in Appendix A. This comes from a feature of the model that each lepton doublet originally couples with only one Higgs doublet. In fact, as shown in Appendix A, we can find the basis of right-handed charged leptons such that the matrix for the Yukawa couplings takes the block diagonal form

$$\tilde{h}_E^{\alpha\beta} L_\alpha \bar{E}'_\beta H_{d_2} + \tilde{h}_E^{33} L_3 \bar{E}'_3 H_{d_3}. \qquad (13)$$

This new basis is consistent with the imposed discrete symmetry. Thus, there appears no additional lepton flavor mixing induced through the light Higgs sector under the supposed conditions.

The down quark sector has the mass mixing with the extra colored fields as

$$(\tilde{Q}_i \ \tilde{g}_3) \begin{pmatrix} (m_d)^{ij} & h_{\tilde{g}}^i v_{1b} \\ h_g^j u' & ku \end{pmatrix} \begin{pmatrix} \bar{\tilde{D}}_j \\ \bar{\tilde{g}}_3 \end{pmatrix}. \qquad (14)$$

This is an important feature of this model. The mixing may be detected as the deviation from the CKM scheme through future experiments for the B meson system. These extra colored fields are also expected to be found at the LHC directly. If each Yukawa coupling

takes appropriate values, these mass matrices (12) and (14) are expected to realize the mass eigenvalues and the flavor mixings. However, since the model can not predict the magnitude and flavor structure of the Yukawa couplings at the present stage, we do not discuss this problem further here.

3.2   Mass and mixing of neutrinos

Next, we proceed to estimate the mass and the mixing of neutrinos in the model. For this purpose, it is useful to investigate the nature of other neutral fermions in this effective model. We have Z_2 even heavy neutral fermions $\tilde{H}_{u_3}$, $\tilde{H}_{d_1}$ and $\tilde{\bar{N}}_3$ in addition to the ordinary left-handed neutrinos $\nu_i (\equiv \tilde{L}_i^0)$. Their masses are induced through the VEV ϕ. This means that their Z_2 odd scalar partners are sufficiently heavy. On the other hand, we have Higgsinos $\tilde{H}_{u_{1,2}}$, $\tilde{H}_{d_{2,3}}$, singlinos $\tilde{S}_{2,3}$, $\bar{N}_{1,2}$, and gauginos $\tilde{W}_3$, $\tilde{B}$, $\tilde{\lambda}_\chi$ as Z_2 odd neutral fermions which constitute a part of neutralinos. Since the scalar partners of the Higgsinos and the singlinos except for H_{u_1} and $\bar{N}_{1,2}$ have the VEVs as discussed before, they can obtain the masses through the effective μ terms and mix with other Z_2 odd neutral fermions including gauginos. The mass matrices of these neutral fermions are given in Appendix B. Since all of these neutralinos can have weak scale masses, the model can satisfy the experimental constraint imposed by the Z^0 invisible width. If we remind that this Z_2 symmetry remains as the exact one even after the symmetry breaking at the weak scale, we find that the left-handed neutrinos ν_i can not mix with these neutralinos. Thus, a dark matter candidate in this model is the lightest neutralino which can have different features from the one in the MSSM, in principle.

The present model contains a heavy Z_2 even neutral fermion $\tilde{H}_{u_3}^0$ and its scalar partner which has no VEV. Here we note that Majorana mass is generated for $\tilde{H}_{u_3}^0$ through the mixing with the heavy fermion $\tilde{\bar{N}}_3$. Since they can couple with left-handed neutrinos ν_i as found in W_3, small neutrino masses and non-trivial neutrino mixing can be induced. If the singlet scalars $S_{2,3}$ obtain the VEVs as shown in eq. (10), the first two terms in the third line of W_3 generate Majorana masses for $\nu_{1,2,3}$ through the seesaw mechanism. Unfortunately, only one nonzero mass eigenvalue can be generated from these contributions. However, we find that $\nu_{1,2}$ can also obtain the radiative mass through one-loop diagrams as shown in Fig. 1. These effects make the model viable for the neutrino mass generation.
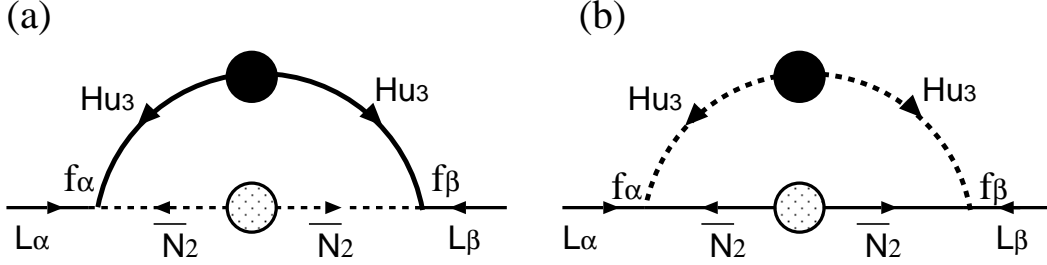
Fig. 1 Diagrams contributing to the neutrino Majorana masses. the bulbs in internal fermion lines of (a) and (b) are induced through the neutral fermion mass matrices in eqs. (23) and (30). On the other hand, the bulbs in internal scalar lines of (a) and (b) is induced by supersymmetry breaking A terms in eqs.(33) and (27), respectively.

The mass matrix for three light Majorana neutrinos $\nu_{1,2,3}$ is expressed by

$$M_\nu = \Lambda_1 \begin{pmatrix} y_1^2 & y_1 y_2 & y_1 \\ y_1 y_2 & y_2^2 & y_2 \\ y_1 & y_2 & 1 \end{pmatrix} + (\Lambda_2^a + \Lambda_2^b) \begin{pmatrix} \bar{f}^2 & \bar{f} & 0 \\ \bar{f} & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{15}$$

where $y_\alpha = f_\alpha u'/f_3 u$ and $\bar{f} = f_1'/f_2'$. The first term is generated through the ordinary seesaw mechanism caused by the heavy neutral fermion $\tilde{H}_{u_3}^0$. The second term is generated through the radiative seesaw mechanism and their relevant diagrams are shown in Fig. 1.[m] The mass scales Λ_1 and $\Lambda_2^{a,b}$ are estimated as

$$\Lambda_1 = \frac{(f_3 u)^2}{M}, \qquad \Lambda_2^a = \frac{(\Lambda_B^a)^2 f_2'^2}{8\pi^2 m_F^a} I\left(\frac{(m_F^a)^2}{(m_B^a)^2}\right), \qquad \Lambda_2^b = \frac{(\Lambda_B^b)^2 f_2'^2}{8\pi^2 m_F^b} I\left(\frac{(m_F^b)^2}{(m_B^b)^2}\right),$$

$$I(x) = \frac{x}{1-x}\left(1 + \frac{x}{1-x}\ln x\right) \tag{16}$$

where M is the effective mass of H_{u_3}, which is generated by the VEV ϕ. In eq. (16), $m_B^a(m_B^b)$ and $\Lambda_B^a(\Lambda_B^b)$ stand for the mass and the mixing shown by the bulbs in the internal scalar lines in Fig. 1(a)(Fig. 1(b)), respectively. The effective mass of the internal fermion is expressed by $m_F^{a,b}$. Detailed discussion on these issues is given in Appendix B. Since both the fermionic component $\tilde{H}_{u_3}$ and the bosonic component H_{u_3} obtain the

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[m]The similar type of radiative neutrino mass generation in supersymmetric model has been discussed in [76, 77]. However, details are different between the present model and the one discussed there. As mentioned in the previous parts, the present extra U(1)s make it possible to solve the μ problem radiatively and also give both the origins of the existence of the singlet chiral superfields and the right-handed neutrino mass scale.
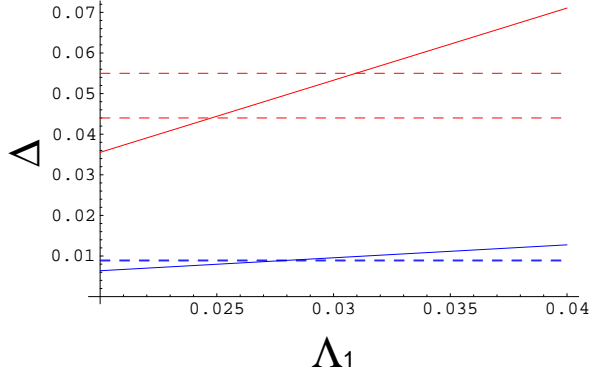
Fig. 2 Mass eigenvalues as functions of Λ_1 in the normal hierarchy case with $\sin\theta_{13} = 0.13$. Red and blue solid lines represent $\Delta_{31} \equiv \sqrt{m_3^2 - m_1^2}$ and $\Delta_{21} \equiv \sqrt{m_2^2 - m_1^2}$, respectively. The regions of $\sqrt{\Delta m_{\text{atm}}^2}$ and $\sqrt{\Delta m_{\text{sol}}^2}$ required by the neutrino oscillation data are plotted by the red and blue dashed lines. All mass units are taken as eV.

large masses due to the VEV ϕ, $m_F^a \gg m_B^a$ is satisfied in Fig. 1(a) and also $m_B^b \gg m_F^b$ in Fig. 1(b). Using this fact, $\Lambda_2^{a,b}$ can be estimated for each case as

$$
\begin{aligned}
\text{(a)} \quad \Lambda_2^a &\simeq \frac{(\Lambda_B^a)^2 f_2'^2}{8\pi^2 m_F^a} \ln\frac{(m_F^a)^2}{(m_B^a)^2} \simeq \frac{f_2'^2 \kappa_2^2 A^3 \mu' v_2 v_{1b}}{8\pi^2 \gamma_1 \phi m_s^4} \ln\left(\frac{(\gamma_1\phi)^2}{m_s^2}\right), \\
\text{(b)} \quad \Lambda_2^b &\simeq \frac{(\Lambda_B^b)^2 f_2'^2 m_F^b}{8\pi^2 (m_B^b)^2} \simeq \frac{f_2'^2 A^5 (\gamma_2 v_2)^2 m_w}{8\pi^2 (\gamma_1 \phi)^4 M_N^3}.
\end{aligned}
\tag{17}
$$

From these expressions, we find that the diagram (a) gives the dominant contribution to the neutrino masses. This scale can be desirable values if the couplings and the soft supersymmetry breaking parameters are fixed appropriately within the reasonable regions. Since the VEV ϕ is sufficiently large as discussed before, we do not need any unnaturally small coupling constants to generate the small neutrino masses. In this point the present model improves the original non-supersymmetric version for the radiative seesaw model [12].

We note that the mass matrix M_ν is expressed by using only 5 independent parameters. This makes the model predictive. If we impose the known experimental data, we can predict a value of $\sin\theta_{13}$. We examine the validity of this neutrino mass matrix by imposing the neutrino oscillation data. We know how neutrinos should mix each other by using the neutrino oscillation data [1–4]. We require that the neutrino mass matrix M_ν

is diagonalized by

$$
U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$
(18)

where U is defined by $U^T M_\nu U = \mathrm{diag}(m_1, m_2, m_3)$. Under this requirement, the mixing angles should satisfy $\sin\theta_{23} \simeq 1/\sqrt{2}$ and $\sin\theta_{12} \simeq 1/\sqrt{3}$. We fix these values just as $1/\sqrt{2}$ and $1/\sqrt{3}$, for simplicity. Since M_ν has only two non-zero mass eigenvalues, we have two possibilities for the mass eigenvalues $m_{1,2,3}$ under this setting: that is, $m_1 = 0$, $m_2 = \sqrt{\Delta m_{\mathrm{sol}}^2}$ and $m_3 = \sqrt{\Delta m_{\mathrm{atm}}^2}$ for normal hierarchy, and $m_3 = 0$, $m_1 = \sqrt{\Delta m_{\mathrm{sol}}^2}$ and $m_2 = \sqrt{\Delta m_{\mathrm{atm}}^2}$ for inverse hierarchy.

We search solutions in which 5 parameters are consistently fixed by applying the neutrino oscillation data to $\Delta m_{\mathrm{sol}}^2$ and $\Delta m_{\mathrm{atm}}^2$ and varying the value of $\sin\theta_{13}$ within the range $0 \le \sin^2 2\theta_{13} < 0.19$ [78]. As a result of this study, we can find consistent solutions only in the normal hierarchy case. Inverse hierarchy seems not to be favored in this model. In Fig. 2, we show an example of the solution with $\sin\theta_{13} = 0.13$. Required values for $\Delta m_{\mathrm{sol}}^2$ and $\Delta m_{\mathrm{atm}}^2$ are plotted as regions sandwiched by two blue and red dashed lines, respectively. In the same figure, eigenvalues m_2 and m_3 are plotted by blue and red solid lines as functions of Λ_1. The figure shows that each solid line crosses the required regions for $\Delta m_{\mathrm{sol}}^2$ and $\Delta m_{\mathrm{atm}}^2$ at the same Λ_1. This means that the model can have the consistent parameter sets for the explanation of the neutrino oscillation data. The fixed values of the 5 parameters in M_ν are determined for this solution. They are listed in Table 2. In the same table, as examples of other typical solutions, we also give the values of these parameters for the different $\sin\theta_{13}$. From these examples, we find that our model can have solutions with very small values of $\sin\theta_{13}$ and also solutions with its present bound value. In the last column of this table, we also show the predicted values of the effective mass $\langle m_{ee} \rangle = |\sum_i U_{ei}^2 m_i|$ which is the measure of the neutrinoless double β decay. These values are two order of magnitude smaller than the present bound. Thus, it seems to be difficult to find the signature of this model in the next generation experiments prepared for the neutrinoless double β decay.

$\sin\theta_{13}$	Λ_1	Λ_2	y_1	y_2	$\bar{f}$	$\langle m_{ee} \rangle$
0.13	0.028	0.371262	0.046855	0.767892	0.59699	3.8×10^{-3}
0.0205	0.023	-0.59241	0.165318	1.2649	0.514604	3.0×10^{-3}
0.2306	0.019	-0.747848	0.580237	1.35967	0.681424	1.5×10^{-5}

Table 2 Numerical values of the parameters for the solutions in the normal hierarchy case. All dimensional parameters are given by the eV unit.

3.3 Other phenomenological features

The model has various features discriminating from the MSSM at TeV regions. Some of them are caused by the existence of extra color triplets and extra Higgs doublets. The former makes it possible to generate two types of μ-terms radiatively through the RGE effects on the soft masses of the singlet scalars. They mix with the down type quarks as mentioned before and we may find these extra color triplets in the LHC experiments [79, 80]. The latter brings special structure for the charged lepton mass matrix and also the extended chargino and neutralino sector through two types of μ-term. We can expect that these features make it possible to distinguish the model from the MSSM. They may be detected in future experiments.

One-loop diagrams similar to the one for the neutrino masses in Fig. 1 might cause additional contributes to other phenomena, which do not appear in the MSSM. In fact, the lepton flavor violating processes such as $\mu \to e\gamma$ are known to give severe constraints on the original non-supersymmetric model for the radiative seesaw [16].[n] Although the small neutrino masses are guaranteed by an extremely small Higgs coupling in the scalar potential in this original model, the existence of the heavy doublet chiral superfield H_{u3} makes the neutrino mass small in the present model. Thus, we need no such a small coupling here. This makes the nature of the new contributions to the lepton flavor violating processes very different in both cases. Since these contributions are sufficiently suppressed due to the heaviness of H_{u_3} in the present model, the dominant constraints from the lepton flavor violating processes appear as the conditions for the ordinary contributions caused by the supersymmetry breaking sector. Thus, the neutrino mass generation is not affected by the FCNC constraints. This is different from the original radiative seesaw

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[n]A possibility to loose this tension in a non-SUSY framework is proposed in [27, 28].

model. Because of the same reason, new contributions to the muon anomalous magnetic moment δa_μ due to the similar one-loop diagrams are also negligible. If we try to explain the presently reported experimental value $\delta a_\mu = (30.2 \pm 8.7) \times 10^{-10}$ [81, 82], we need to find its origin in the contributions expected also in other supersymmetric models.

Finally, we comment on a dark matter candidate. Our model has the exact Z_2 symmetry, which can be identified with the R parity. Since the even parity of this Z_2 symmetry are assigned to the SM contents, dark matter is consider to be the lightest neutral field with the odd parity. Thus, the candidate is expected to be the lightest neutralino as mentioned before. In the present model, there are new neutralino components other than the MSSM ones. Since the lightest one is not related to the one-loop neutrino mass generation discussed before, its abundance has no direct relation with both the neutrino mass generation and the constraint from the lepton flavor violating processes such as $\mu \to e\gamma$. The nature of the lightest neutralino is determined by their mass matrix $\mathcal{M}_\mathcal{N}$ given in Appendix B. Its detailed analysis is beyond the scope of this paper. Here, we only point out an interesting possibility on this issue, especially, the relation between the dark matter relic abundance and the PAMELA anomaly [83] which shows the positron excess at 10 GeV-100 GeV in the cosmic ray from the galactic center.

We consider a case in which a neutralino dominated by $\tilde{N}_1$ and $\tilde{H}_{u_1}$ or $\tilde{N}_2$ and $\tilde{H}_{u_2}$ is the lightest one. In that case their nature as the dark matter is completely different from that of the dark matter candidate in the MSSM. Since such states have the Yukawa couplings $\kappa_1 \tilde{\bar{N}}_1 \tilde{H}_{u_1} H_{d_3}$, $\kappa_2 \tilde{\bar{N}}_2 \tilde{H}_{u_2} H_{d_3}$ and also $\mathrm{U}(1)_\chi$ gauge interaction, they are expected to annihilate through these interactions into quarks and leptons. Since H_{d_3} couples only with leptons in the light fields as found from W_3, the annihilation of the dark matter through the s-channel exchange of the neutral Higgs scalar $H_{d_3}^0$ can produce positrons but no antiprotons in the final states.º On the other hand, since quarks and leptons have non-zero $\mathrm{U}(1)_\chi$ charge, the dark matter can annihilate to both quarks and leptons through this gauge interaction. These aspects may allow us to understand the discrepancy between the values of the thermally averaged annihilation cross section $\langle \sigma v \rangle$ required to explain the relic abundance and the PAMELA anomaly, respectively. In order to explain this reason, we suppose that the model parameters can be arranged so that the $\mathrm{U}(1)_\chi$

ºIt should be noted that $h_{\bar{g}}^i Q_i \bar{g}_3 H_{d_3}$ can not contribute to the dark matter annihilation since $\bar{g}_3$ is much heavier than the supposed dark matter.

interaction is relevant only to the determination of the relic abundance of the dark matter but the Yukawa interactions are only related to the PAMELA anomaly. If the annihilation cross section due to the $U(1)_\chi$ interactions satisfies $\langle \sigma v \rangle \sim 3 \times 10^{-26}$ cm^3/sec at the freeze-out time of the dark matter, it is known to be suitable for the explanation of the relic abundance but smaller than the required one for the PAMELA anomaly by two or three order of magnitude. However, if the dark matter mass is almost equal to half of the mass eigenvalue of the neutral Higgs state dominated by $H_{d_3}^0$, the Breit-Wigner enhancement may make this annihilation cross section much larger in the present Galaxy [84,85]. In that case, the final states of the annihilation is mainly composed of leptons. This may allow the model to give a consistent explanation for this huge boost factor problem. Since the dark matter can not be so heavy in the present model as found from the neutralino mass matrix (30), we need origins other than the dark matter annihilation for the explanation of the excess of positron and electron flux at higher energy regions observed in the Fermi-LAT experiment [86]. We would like to discuss this issue quantitatively elsewhere.

4   Summary

Present experimental data on dark matter and neutrino masses impose us to extend the SM. In this paper we have considered a new possibility of such extensions in the framework of a supersymmetric model with an extra U(1) symmetry at TeV regions, which is constructed on the basis of E_6. In the ordinary E_6 framework, unless the fields of higher dimensional representations are introduced, only a unique example of the extra U(1) has been known to be consistent with large right-handed Majorana neutrino masses. In that case the small neutrino mass generation can be considered on the basis of the seesaw mechanism. In other types of low energy extra U(1) symmetry in the E_6 framework, however, it is difficult to make the right-handed neutrinos heavy enough keeping the consistency with this TeV scale U(1) symmetry. Thus, the neutrino masses can not be small enough naturally as long as we follow the usual field assignment. Although the extra U(1) symmetry in $O(1)$ TeV regions gives an elegant solution for the μ problem in the MSSM, this solution can not be consistent with the small neutrino mass generation based on the seesaw mechanism except for the unique case mentioned above.

In this paper we have proposed a scenario in which a new type of extra U(1) symmetry

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in E_6 may give a consistent explanation for both the small neutrino mass generation and the μ problem. By embedding the MSSM fields in the fundamental representation **27** in the different way among the generations, we have shown that both of them can be consistently explained. The small neutrino masses are generated by both the seesaw mechanism due to a heavy SU(2) doublet fermion and also one-loop effects similar to the radiative seesaw in the non-supersymmetric model. Since neutrino mass matrix has the constrained texture with the restricted number of parameters, the model is predictive in the neutrino sector. Our numerical study shows that the model can be consistent with all neutrino oscillation data. For that parameter set $\sin\theta_{13}$ can be predicted. We have given such examples. The model has other interesting phenomenological features, that is, the gauge coupling unification expected at a GUT scale, the existence of several extra fields which may be detected in the LHC experiment and others, and also the dark matter candidate which may have different features from the one of the MSSM. These aspects seems to make the model interesting and also deserve further study.

Appendix A

The charged lepton mass matrix in eq. (12) is composed of Yukawa couplings with two Higgs doublets. However, if Yukawa coupling constants are assumed to satisfy simple relations, each mass eigenstate couples with only one Higgs doublet as shown in eq. (13). This guarantees to bring no additional origin for the lepton flavor violating processes. As such conditions, we may adopt

$$\sum_{\beta=1}^{3} h_E^{\alpha\beta} h_E^{3\beta} = 0 \qquad (\alpha = 1, 2). \tag{19}$$

In this case, we can easily find that eq. (13) is realized, if we take a new basis for the right-handed charged leptons such as $\bar{E}' = V\bar{E}$ where V is defined as

$$V = \begin{pmatrix} \frac{h_E^{32}}{\xi_1} & -\frac{h_E^{31}}{\xi_1} & 0 \\ \frac{h_E^{31} h_E^{33}}{\xi_1 \xi_2} & \frac{h_E^{32} h_E^{33}}{\xi_1 \xi_2} & -\frac{\xi_1}{\xi_2} \\ \frac{h_E^{31}}{\xi_2} & \frac{h_E^{32}}{\xi_2} & \frac{h_E^{33}}{\xi_2} \end{pmatrix}, \tag{20}$$

and $\xi_n = (\sum_{\alpha=1}^{n+1} (h_E^{3\alpha})^2)^{1/2}$. Yukawa coupling constants in this new basis are expressed as

$$\tilde{h}_E^{11} = (h_E^{11} h_E^{32} - h_E^{12} h_E^{31})/\xi_1, \qquad \tilde{h}_E^{12} = -h_E^{13} \xi_2/\xi_1,$$

$$\tilde{h}_E^{21} = (h_E^{21} h_E^{32} - h_E^{22} h_E^{31})/\xi_1, \qquad \tilde{h}_E^{22} = -h_E^{23} \xi_2/\xi_1, \qquad \tilde{h}_E^{33} = \xi_2. \tag{21}$$

Appendix B

In this appendix we address both masses and mixings of the fields which play important roles in the neutrino mass generation shown by Fig. 1. They are expressed by m_F, m_B and Λ_B in the formulas for Λ_2. Here we omit the suffices a and b which are written in the text. In Fig. 1 m_F and Λ_B are drawn by the bulbs.

There are heavy colorless chiral superfields, which obtain masses through the couplings with $\mathcal{A}_7$ and $\bar{\mathcal{A}}_7$ as discussed in the text. Their fermionic components are Z_2 even as the ordinary quarks and leptons. The effective superpotential for these heavy chiral superfields at the weak scales are found to be given by

$$W_H = \gamma_1 \phi H_{u_3} H_{d_1} + \gamma_2 v_2 H_{d_1} \bar{N}_3 + \frac{1}{2} M_N \bar{N}_3^2, \tag{22}$$

where we list up dominant terms only and $M_N = O(\phi^2/M_{\rm pl})$. The first two terms come from the last line of the superpotential W_2. A mass term for $\bar{N}_3$ is induced through the interaction in eq. (8). Thus, the mass matrix for the neutral fermionic components of H_{u_3}, H_{d_1}, $\bar{N}_3$ can be expressed on the $(\tilde{H}^0_{u_3}, \tilde{H}^0_{d_1}, \tilde{\bar{N}}_3)$ basis as

$$\mathcal{M}_H = \begin{pmatrix} 0 & \gamma_1\phi & 0 \\ \gamma_1\phi & 0 & \gamma_2 v_2 \\ 0 & \gamma_2 v_2 & M_N \end{pmatrix}. \tag{23}$$

As found from the couplings of $\tilde{H}^0_{u_3}$ with ν_i in the superpotential W_3, $\tilde{H}^0_{u_3}$ plays the similar role in the neutrino mass generation to the one of the right-handed neutrinos in the ordinary seesaw mechanism. It also contribute to the one-loop diagram for the neutrino mass generation in Fig. 1. If we define a mixing matrix by $(V_H)^T \mathcal{M}_H V_H = \mathcal{M}_H^{\rm diag}$, we find that the mass eigenvalues and the mixing matrix V_H are estimated as

$$\mathcal{M}_H^{\rm diag} \simeq {\rm diag}(\gamma_1\phi,\ \gamma_1\phi,\ M_N), \qquad V_H \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}}e^{i\frac{\pi}{2}} & \frac{1}{\sqrt{2}}e^{i\frac{\pi}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{24}$$

where we use the relation $\phi,\ M_N \gg v_2$ to derive these results. If we express Λ_2 in eq. (16) as $\Lambda_2 = f(m_F, m_B)$, Λ_2 can be written by using the mass eigenstates derived above as

$$\Lambda_2 = \sum_{a=1}^{3} \{(V_H)_{1a}\}^2 f(M_a, m_B) = f(\gamma_1\phi, m_B). \tag{25}$$

Thus, the effective mass m_F of the internal fermion in Fig. 1(a) can be estimated as $m_F \sim \gamma_1\phi$.

Next, we represent scalar partners of these chiral superfields as $\Phi = (H^0_{u_3}, H^0_{d_1}, \bar{N}_3)$ and define their mass terms by

$$-\mathcal{L}_\Phi = \frac{1}{2}\left(\Phi^\dagger \mathcal{M}_B^2 \Phi + \Phi^T \mathcal{M}_m^2 \Phi\right) + {\rm h.c.}, \tag{26}$$

where these mass matrices can be expressed as

$$\mathcal{M}_B^2 = \begin{pmatrix} \gamma_1^2\phi^2 + m_s^2 & 0 & \gamma_1\gamma_2\phi v_2 \\ 0 & \gamma_1^2\phi^2 + \gamma_2^2 v_2^2 + m_s^2 & \gamma_2 v_2 M_N \\ \gamma_1\gamma_2\phi v_2 & \gamma_2 v_2 M_N & M_N^2 + \gamma_2^2 v_2^2 + m_s^2 \end{pmatrix},$$

$$\mathcal{M}_m^2 = \begin{pmatrix} 0 & A\gamma_1\phi & 0 \\ A\gamma_1\phi & 0 & A\gamma_2 v_2 \\ 0 & A\gamma_2 v_2 & AM_N \end{pmatrix}. \tag{27}$$

In these mass matrices we introduce the supersymmetry breaking universal soft scalar masses m_s^2 and also the universal soft supersymmetry breaking parameter A for the scalar trilinear couplings. Since ϕ constitutes a D-flat direction for $U(1)_\psi$, there are no D-term contributions to the scalar masses. By using these results, the effective mass m_B and the mixing Λ_B appeared in the internal scalar line of Fig. 1(b) are estimated as

$$m_B^2 \simeq (\gamma_1 \phi)^2, \qquad \Lambda_B^2 \simeq \frac{A^5 (\gamma_2 v_2)^2}{M_N^3 (\gamma_1 \phi)^2}, \tag{28}$$

where we again take account of $\phi \gg v_2, m_s$ to derive these results.

The model contains a lot of Z_2 odd neutral fermions and their Z_2 even scalar partners. One-loop diagrams for the neutrino mass generation in Fig. 1 include chiral superfield $\bar{N}_2$ which is a member of them. Thus, the mixings of its fermionic partner with other Z_2 odd fermions are crucial for the estimation of this diagram. These light Z_2 odd neutral fermions include the SU(2) gaugino $\tilde{W}_3$, the $U(1)_Y$ gaugino $\tilde{B}$, the $U(1)_\chi$ gaugino $\tilde{\lambda}_\chi$ and several fermionic components of chiral superfields. Relevant terms in the superpotential W_3 and W_2^{NR} in eq. (4) are

$$W_{\mathcal{N}} = \lambda_1 S_2 H_{u_2} H_{d_2} + \lambda_2 S_3 H_{u_2} H_{d_3} + \kappa_1 \bar{N}_1 H_{u_1} H_{d_3} + \kappa_2 \bar{N}_2 H_{u_2} H_{d_3} + \kappa_3 \epsilon^2 \phi H_{u_1} H_{d_2}, \quad (29)$$

where we introduce a new coupling constant κ_3. If we take a basis for these neutral fermions as

$$\tilde{\mathcal{N}} = (\tilde{W}_3, \tilde{B}, \tilde{\lambda}_\chi, \tilde{H}_{u_2}, \tilde{H}_{d_2}, \tilde{H}_{d_3}, \tilde{S}_2, \tilde{S}_3, \tilde{H}_{u_1}, \tilde{N}_1, \tilde{N}_2),$$

their tree-level mass matrix $\mathcal{M}_{\mathcal{N}}$ can be expressed as

$$\begin{pmatrix}
M_2 & 0 & 0 & -\frac{g_2 v_2}{\sqrt{2}} & \frac{g_2 v_{1a}}{\sqrt{2}} & \frac{g_2 v_{1b}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\
0 & M_1 & 0 & \frac{g_1 v_2}{\sqrt{2}} & -\frac{g_1 v_{1a}}{\sqrt{2}} & -\frac{g_1 v_{1b}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & M_\xi & \frac{q_2 g_\xi v_2}{\sqrt{2}} & \frac{q_1 g_\xi v_{1a}}{\sqrt{2}} & \frac{q_1 g_\xi v_{1b}}{\sqrt{2}} & \frac{q_S g_\xi u}{\sqrt{2}} & \frac{q_S g_\xi u'}{\sqrt{2}} & 0 & 0 & 0 \\
-\frac{g_2 v_2}{\sqrt{2}} & \frac{g_1 v_2}{\sqrt{2}} & \frac{q_2 g_\xi v_2}{\sqrt{2}} & 0 & \mu & \mu' & \lambda_1 v_{1a} & \lambda_2 v_{1b} & 0 & 0 & \kappa_2 v_{1b} \\
\frac{g_2 v_{1a}}{\sqrt{2}} & -\frac{g_1 v_{1a}}{\sqrt{2}} & \frac{q_1 g_\xi v_{1a}}{\sqrt{2}} & \mu & 0 & 0 & \lambda_1 v_2 & 0 & \kappa_3 \epsilon^2 \phi & 0 & 0 \\
\frac{g_2 v_{1b}}{\sqrt{2}} & -\frac{g_1 v_{1b}}{\sqrt{2}} & \frac{q_1 g_\xi v_{1b}}{\sqrt{2}} & \mu' & 0 & 0 & 0 & \lambda_2 v_2 & 0 & 0 & \kappa_2 v_2 \\
0 & 0 & \frac{q_S g_\xi u}{\sqrt{2}} & \lambda_1 v_{1a} & \lambda_1 v_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{q_S g_\xi u'}{\sqrt{2}} & \lambda_2 v_{1b} & 0 & \lambda_2 v_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \kappa_3 \epsilon^2 \phi & 0 & 0 & 0 & 0 & \kappa_1 v_{1b} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_1 v_{1b} & 0 & 0 \\
0 & 0 & 0 & \kappa_2 v_{1b} & 0 & \kappa_2 v_2 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}. \tag{30}$$

Each neutralino component $\tilde{\mathcal{N}}_n$ is related to the mass eigenstates $\tilde{\chi}_a$ through

$$\tilde{\mathcal{N}}_n = \sum_a (V_N)_{na} \tilde{\chi}_a, \tag{31}$$

where V_N is the mixing matrix which diagonalizes the neutralino mass matrix $\mathcal{M}_N$ defined in eq. (30) as $V_N^T \mathcal{M}_N V_N = \mathrm{diag}(\tilde{M}_1, \tilde{M}_2, \cdots, \tilde{M}_{11})$. In this neutralino case, Λ_2 can be expressed as

$$\Lambda_2 = \sum_{a=1}^{11} \left\{ (V_N)_{9a} \right\}^2 f(\tilde{M}_a, m_B). \tag{32}$$

Since details are dependent on a lot of parameters, we can not give analytic expression for the effective mass m_F. However, it is obvious that m_F takes a weak scale value m_w. This rough estimation is enough for the present purpose.

In order to estimate m_B and Λ_B for the scalar partners of these fermions, we need to take account of the couplings $\lambda_2 S_3 H_{u_2} H_{d_3}$ and $\kappa_2 \bar{N}_2 H_{u_2} H_{d_3}$ in $W_{\mathcal{N}}$. If we adopt a basis as $\Phi = (H_{u_2}^0, H_{d_3}^0, S_3, \bar{N}_2)$ and express the mass terms by eq. (26), $\mathcal{M}_B^2$ and $\mathcal{M}_m^2$ in this case can be written as

$$\mathcal{M}_B^2 = \begin{pmatrix} \mu'^2 + \bar{\lambda}_2^2 v_{1b}^2 + m_s^2 & 0 & 0 & 2\kappa_2 \mu' v_2 \\ 0 & \mu'^2 + \bar{\lambda}_2^2 v_2^2 + m_s^2 & 0 & 2\kappa_2 \mu' v_{1b} \\ 0 & 0 & \lambda_2^2 \bar{v}_b^2 + m_s^2 & \lambda_2 \kappa_2 \bar{v}_b^2 \\ 2\kappa_2 \mu' v_2 & 2\kappa_2 \mu' v_{1b} & \lambda_2 \kappa_2 \bar{v}_b^2 & \kappa_2^2 \bar{v}_b^2 + m_s^2 \end{pmatrix},$$

$$\mathcal{M}_m^2 = \begin{pmatrix} 0 & A\mu' & A\lambda_2 v_{1b} & A\kappa_2 v_{1b} \\ A\mu' & 0 & A\lambda_2 v_2 & A\kappa_2 v_2 \\ A\lambda_2 v_{1b} & A\lambda_2 v_2 & 0 & 0 \\ A\kappa_2 v_{1b} & A\kappa_2 v_2 & 0 & 0 \end{pmatrix}, \tag{33}$$

where $\bar{v}_b^2 = v_2^2 + v_{1b}^2$ and $\bar{\lambda}^2 = \lambda_2^2 + \kappa_2^2$. Although there are D-term contributions to $\mathcal{M}_B^2$, they are not written explicitly here. From these mass matrices, the effective mass m_B^2 and the mixing Λ_B^2 in Fig. 1(a) are approximately estimated as

$$m_B^2 \simeq m_s^2, \qquad \Lambda_B^2 = \frac{\kappa_2^2 A^3 \mu' v_2 v_{1b}}{m_s^4}. \tag{34}$$

We find that both of these take values of order of the weak scale.

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